CHAPTER EIGHT

LOGARITHM

Introduction:

* In $\log_b N$, N is referred to as the number and b is referred to as the base.

* The logarithm of a positive number N to a given base b, is the power to which b must be raised so as to be equal to N.

* For example, if $\log_x{}^y = k$, then $x^k = y$.

* If $\log_3{}^9 = 2 \Rightarrow 3^2 = 9$.

* Also, $\log_2{}^{16} = 4 \Rightarrow 2^4 = 16$

* Since $\log_4{}^{16} = 2$, then $4^2 = 16$

(Q1) Determine the value of x, given that

a) $\text{Log}_5{}^{25} = x$ (b) $\log_2{}^4 = 4$

(c)$\log_2{}^{32} = x$ (d) $\log_5{}^{125} = x$

(e) $\log_5{}^{625} = x$ (f) $\log_3{}^{81} = x$.

Soln:

(a) Since $\text{Log}_5{}^{25} = x$, then $5^x = 25 \Rightarrow 5^x = 5^2$

$\Rightarrow x = 2$.

(b) if $\log_2{}^4 = x$, then $2^x = 4$

$\Rightarrow 2^x = 2^2 \Rightarrow x = 2$.

(c) Since $\log_2{}^{32} = x$, then $2^x = 32$

$\Rightarrow 2^x = 2^5 \Rightarrow x = 5$.

(d) Since $\log_5^{125} = x$, then $5^x = 125 \Rightarrow 5^x = 5^3 \Rightarrow x = 3$.

(e) If $\log_5^{625} = x$, then $5^x = 625 \Rightarrow 5^x = 5^3 \Rightarrow x = 3$.

(Q2) Determine the value of y if

(a) $\log_y^4 = 2$ (b) $\log_y^{16} = 2$

(c) $\log_y^{36} = 2$ (d) $\log_y^{81} = 4$

(e) $\log_y^{64} = 3$ (f) $\log_y^{27} = 3$.

Soln:

Since $\log_y^4 = 2$, then $y^2 = 4$

$\Rightarrow y^2 = 2^{2\,=} \Rightarrow y = 2$.

a). Since $\log_y^{16} = 2$, then $y^2 = 16$

$\Rightarrow Y^2 = 4^2 \Rightarrow y = 4$.

a) Since $\log_y^{36} = 2$, then $y^2 = 36$

$\Rightarrow y^2 = 6^2 \Rightarrow y = 6$.

(d)Since $\log_y^{81} = 4$, then $y^4 = 81$

$\Rightarrow y^4 = 3^4 \Rightarrow y = 3$.

(e) Since $\log_y^{64} = 3$, then $y^3 = 64$

$\Rightarrow y^3 = 4^3 \Rightarrow y = 4$.

(f) $\log_y^{27} = 3$, then $y^3 = 27$

$\Rightarrow y^3 = 3^3 \Rightarrow y = 3$.

N/B: (1) If no base is written or indicated, then we are dealing in base 10.

* For example, $\log 10 = \log_{10}^{10}$ and $\log 8 = \log_{10}^{8}$.

* (2) If the value of the number and the base are the same, then the value of the log is 1.

* For example, $\log_{10}{}^{10}$ = 1 and $\log_2{}^2$ = 1.

* Also $\log_5{}^5$ =1 and $\log_4{}^4$ = 1.

Determination of values of logarithm:

*This can be done by either using a four figure table or a scientific calculator.

Using the four figure table:

*In this case, the decimal point must be after the first number.

* If this is not so, then it must be brought after the first number.

* If this point has to be moved or shifted once towards the left, then the character is 1.

* If it is moved twice, or by two steps, then the characteristic is two.

* If it is moved thrice or by three steps toward the left, then the characteristic becomes three and so on.

* If the decimal point is already after the first number, then there is no movement or shifting of this point, and the characteristic is zero.

(Q1) Determine the characteristic of each of these numbers:

(a) 2.45 (b)3.817

(c)24.5 (d)388.5

(e)24 (f)345

(g)2401 (h)73105

(i)4445.8 (j)3000.43

Soln:

(a) In 2.45, the characteristic is 0, since the point is already after the first number.

(b)Also in 3.817, the characteristic is zero.

(c) 24.5. In this case, the characteristic is 1, since the point has to be shifted one step to the left, in order to be after the first number.

(d) 388.5. In this case, the characteristic is 2, since the point must be shifted two steps left in order to appear after the first number.

(e)24 = 24.0. In this case, the characteristic is 1.

(f) 345 = 345.0. In this case, the characteristic is 2.

(g) 2401 = 2401.0. In this case, the characteristic is 3, since the point must be shifted three steps left, in order to appear after the first number.

(h) 73105 = 73105.0. In this case, the characteristic is 4.

(i) 4445.8. The characteristic is 3.

(j)3000.43. The characteristic is 3.

N/B:

- On the other hand, if the decimal point has to be moved once toward the right before it comes after the first number, then the characteristic is -1.

- If it is moved twice or two steps toward the right, then the characteristic is -2.

- If this movement towards the right is by three steps, then the characteristic is -3.

(Q2) Determine the characteristic of each of these numbers:

(a) 0.24 (b) 0.00789

(c) 0.0005 (c) 0.00085

Soln:

(a) 0.24. The characteristic is -1 in this case, since the point has to be moved one step to the right, in order to appear after the first number.

(b) 0.00789. The characteristic is - 3, since the point has to be shifted three steps to the right, in order to appear after the first number.

(c) 0.0005. The characteristic is - 4.

(d) 0.00085. The characteristic is - 4.

- In the determination of the values of the given logarithm using the table, the characteristic is first determined before the actual value of the log is determined from the table.

- In certain cases, what is referred to as differences may arise.

- The values of these differences which are found at the extreme right hand side of the table, must be added to the values of the logarithm to get our final value.

(Q3) Determine the value of each of the following:

(a) log 0.451 (b) log 0.2453

(c) log 0.245 (d) log 0.2453

(e) log 0.4569 (f) log 0.0171

(g) log 0.01719 (h) log 0.00028

(i) log 0.0002865 (j) log 0.03

(k) log 0.008 (l) log 0.0000895

(m) log 0.0000821368

Soln:

(a) In log 0.451, the characteristic is -1.

- We then determine the value of log 45 under 1, which is = 6542.

=> log 0.451 = -1.6542.

(b) In log 0.45, the characteristic is -1. Since log 0.45 = log 0.450, we determine the value of log 45 under 0 which = 6532.

The value of log 0.45 = - 1.6532.

(c) The characteristic of log 0.245 = -1.

Log 24 under 5 = 3892.

=> The value of log 0.245 = -1.3892.

(d) In Log 0.2453, the characteristic is -1.

- We then determine the value of log 24 under 5, and add to it the value of difference 3.

- Log 24 under 5 = 3892 and the value of the difference 3 in this case = 5.

3892 + 5 = 3897.

=> log 0.2453 = -1.3897.

(e) In log 0.4569, the characteristic = -1.

Log 45 under 6 = 6590 and the difference 9 = 9.

6590 + 9 = 6599.

Log 0.4569 =-1.6599.

(f) In log 0.0171, the characteristic = - 2.

Log 17 under 1 = 2330.

=> log 0.0171 = -2.2330.

(g) In log 0.01719, the characteristic = -2.

log 17 under 1 = 2330 and its difference 9 = 22.

2330 + 22 = 2352.

=> log 0.01719 = -2.2352.

(h) In log 0.00028 which is the same as log 0.000280, the characteristic = - 4 and log 28 under 0 = 4472.

=> log o.ooo28 = - 4.4472.

(i) In log 0.0002865, the characteristic = - 4.

Log 28 under 6 = 4564 and its difference 5 = 8.

4564 + 8 = 4572.

Log 0.0002865 = - 4.4572.

(j) log 0.03 = log 0.0300 and the characteristic = -2.

log 30 under 0 = 4771.

=> log 0.03 = -2.4771.

(k) log 0.008 = log 0.00800 and the characteristic= -3.

Log 80 under zero = 9031.

=> log 0.008 = -3.9031.

(l) In log 0.0000895, the characteristic = -5.

log 89 under 5 = 9518.

=> log 0.0000895 = -5.9518.

(m) In log 0.0000821368, the characteristic = -5.

Log 82 under 1 = 9145 and the difference 3 = 2.

9145 + 2 = 9147.

=> Log 0.0000821364 = -5.9147.

N/B: In log 0.000821368, we only consider log 82 under 1 difference 3.

(Q4) Determine the value of the following:

(a) log 45.1 (b) log 4.51

(c) Log 488 (d) log 4.88

(e) log 4883 (f) log 20.1

(g) log 200.54 (h) log 3.216

(i) log 89668 (j) log 341.67

(k) log 453816 (l) log 4553. 29

Soln:

(a) In log 45.1, the characteristic = 1, and log 45 under 1 = 6542.

=> Log 45.1 = 1.6542.

(b) In 4.51, the characteristic = 0 and log 45 under 1 = 6542.

=> log 4.51 = 0.6542.

(c) In log 488, the characteristic = 2, and log 48 under 8 = 6884.

=> Log 488 = 2.6884.

(d) In log 4.88, the characteristic = 0 and log 48 under 8 = 6884.

=> log 4.88 = 0.6884.

(e) In log 4883, the characteristic = 3.

Log 48 under 8 = 6884 and its difference 3 = 3.

6884 + 3 = 6887.

=> log 4883 = 3.6887

(f) In log 20.1, the characteristic = 1 and log 20 under 1= 3032.

=> log 20.1 = 1.3032.

(g) In log 200.54, the characteristic = 2.

Log 20 under 0 = 3010 and the difference 5 = 11.

3010 + 11 = 3021.

Log 200.54 = 2.3021.

(h) In log 3.216, the characteristic = 0.

Log 32 under 1 = 5065 and the difference 6 = 8.

5065 + 8 = 5073

=> Log 3.216 = 0.5073.

(i) In Log 89668, the characteristic = 4.

Log 89 under 6 = 9523 and the difference 6 = 3.

9523 + 3 = 9526.

=> Log 89668 = 4.9526.

(j) In log 341.67, the characteristic = 2.

Log 34 under 1 = 5328 and the difference 6 = 8.

5328 + 8= 5336.

=> log 341.67 = 2.5336.

(k) Log 453816 = log 453816.0. The characteristic = 5.

Log 45 under 3 = 6561 and the difference 8 = 8.

6561 + 8 = 6569.

Log 453816 = 5.6569.

Determination of the value of logarithm using the calculator:

- If the number has a characteristic which is either zero or positive, then the calculator can be used to determine the value of the logarithm directly or straight away.

- For example, to determine log 1.675 or log 72.1, simply, press log followed by the number.

- If the number has a negative characteristic, we first write down this characteristic.

- We then press log followed by the number which comes after the decimal point and this will give us a number in the form of a decimal.

- The number after the decimal point is approximated and written after the characteristic.

- But a decimal point must first be brought after the characteristic.

- For example, to determine log 0.2715, we first write down the characteristic which is -1.

- The value of log 2715 according to the calculator is 3.434.

- The figure after the decimal point which can be approximated to 434 is then written after the -1.

- But first bring a decimal point after the characteristic.

- Therefore log 0.2715 = -1.434.

- To determine the value of log 0.04343, the characteristic is -2 and log 4343 = 3.637789829.

- The value after the decimal point can be approximated to 6378.

- Therefore log 0.04343 = -2.6378.

Determination of antilog:

- To determine the antilog of a given number which is always in the decimal form using a calculator, you must first press shift or drag followed by log and then the number which comes after the decimal point.

- For example, to get the antilog of 2.752, first press shift, followed by log, and then 0.752.

- This will give us 5.649.

- Also to get antilog -1.6723, press shift , followed by log and then 0.6723.

- This will give us 4.7022.

- If log x = 0.275, then x = antilog 0.275×10^0, i.e. the number after the decimal point times 10 raised to the digit before the point.

- If log x = -3.214, then x = antilog 0.214×10^{-3}

- If log x = 2.823, then x = antilog 0.823×10^2.

- If log x = -2.430 , then x = antilog 0.430×10^{-2}.

-If log x = 5.720, then x = antilog 0.720×10^5

- If log x = -5.360, then x = antilog 0.360×10^{-5}

- If log x = 2.0371, then x = antilog 0.0371×10^2,

- If x = 0.0241, then x = antilog $0.0241 \times 10^0 = 0.0241 \times 1 = 0.0241$.

N/B:

$\log_{10}(a \times b) = \log_{10} a + \log_{10} b$

=> $\log (a \times b) = \log a + \log b$

Example(1)

$\log_{10} (371 \times 4211) = \log_{10} 371 + \log_{10} 4211$

Example (2)

$\log(0.3741 \times 91.7) = \log 0.3741 + \log 91.7$.

Example (3)

$\log(0.721 \times 0.0043)$

$= \log 0.721 + \log 0.0043$

- Any number raised to the power zero = 1

=> $6^0 = 1$ and $2^0 = 1$.

(Q1) Determine value of 37.1×4481.

Soln:

Let $x = 37.1 \times 4481$

Taking log of both sides => $\log x = \log(37.1 \times 4481)$

=> $\log x = \log 37.1 + \log 4481$

=> $\log x = 1.5693 + 3.6514$

=> $\log x = 5.2207$

=> $x = \text{antilog } 0.2207 \times 10$

=> $x = 1.66226 \times 10^5$

=> $x = 166226$.